

Mutual Information Analysis: How, When and Why?

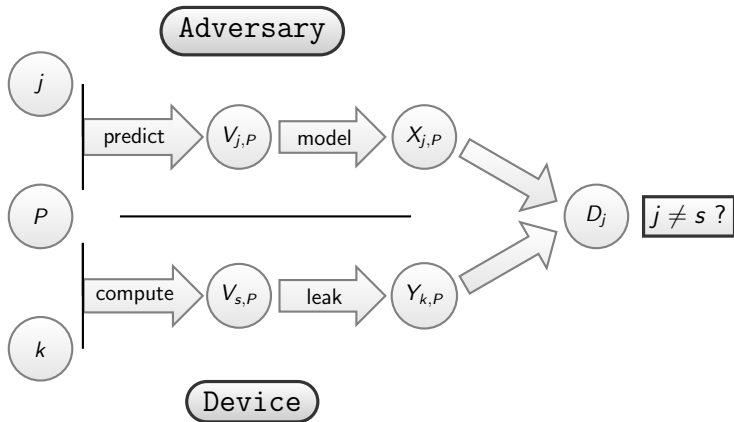
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Université catholique de Louvain

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Side-channel analysis



Classical attacks

Classical solutions in non profiled SCA:

- Kocher's original DPA, at Crypto 1999
- Correlation attacks, at CHES 2004

So, what to do?

$$X_0 =$$

0	1	2	3	4
y_0	y_1	y_3	y_7	y_{15}
...	y_2	y_5	y_{11}	...
	y_4	y_6	y_{13}	
	y_8	y_9	y_{14}	
	...	y_{10}	...	
		y_{12}		
		...		

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	...	y_{11}	...	
		y_{14}		
		...		

Pearson's correlation coefficient

Measure of *linear* dependence between r.v.'s X and Y .

$$\rho(X, Y) = \frac{\text{cov}(X, Y)}{\sigma_X \cdot \sigma_Y} = \frac{E[XY] - E[X] \cdot E[Y]}{\sigma_X \cdot \sigma_Y}.$$

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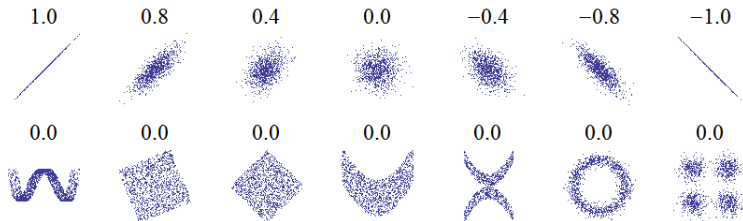
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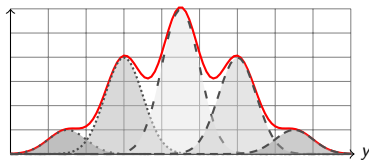
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So, what to do?

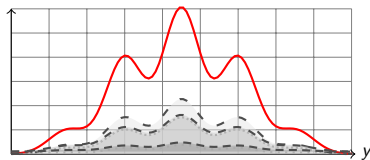
— $\Pr[Y = y]$ \cdots $x = 0$
 $- \cdot$ $x = 2$ $--$ $x = 3$
 $\Pr[Y = y, X = x]$



$$X_0 =$$

0	1	2	3	4
y_0	y_1	y_3	y_7	y_{15}
\dots	y_2	y_5	y_{11}	\dots
	y_4	y_6	y_{13}	
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	\dots	y_{10}	\dots	
		y_{12}		
		\dots		

\cdots $x = 1$
 $--$ $x = 4$
 $\Pr[Y = y, X = x]$



$$X_1 =$$

0	1	2	3	4
y_7	y_0	y_1	y_5	y_6
\dots	y_2	y_3	y_8	\dots
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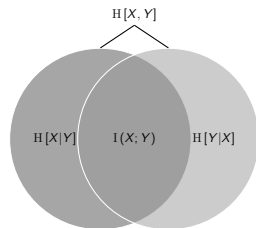
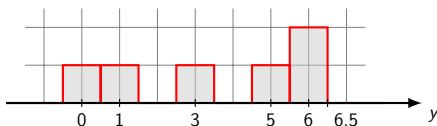
So, what to do?

- 1 Estimate the probability density of the leakages
- 2 Test for a dependence between X and Y

Mutual information Analysis

Introduced at CHES 2008 by Gierlichs & *al.*

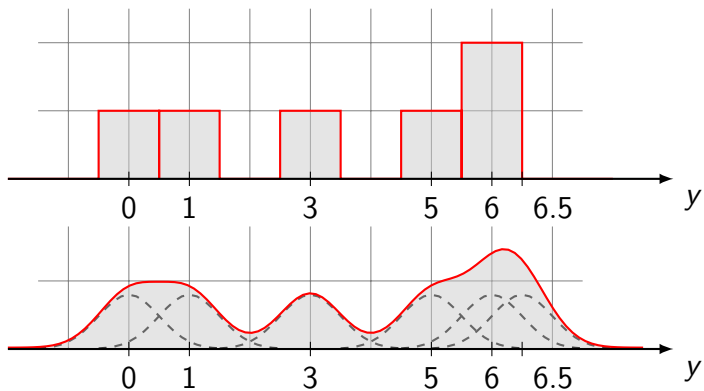
Aims at genericity: as little assumptions as possible about the leakage



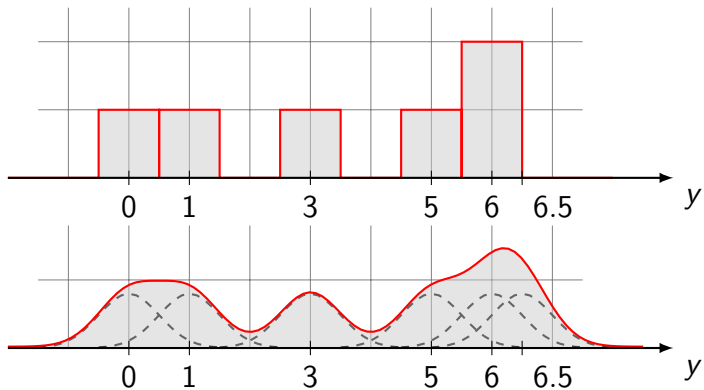
Outline

- 1 How to use MIA: the information theoretic toolbox
- 2 When to use it: MIA versus correlation
- 3 Why to use it: MIA as an evaluation metric

1 Estimation: Non-parametric methods



1 Estimation: Non-parametric methods



Well, non-parametric... bin width and bandwidth to choose

Information theoretic definitions

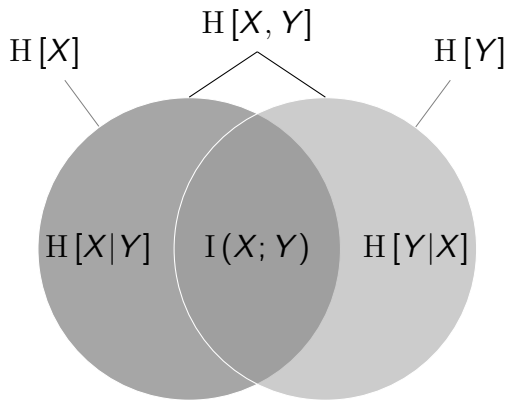
- Shannon's entropy, a measure of information

$$H[X] = - \sum_{x \in \mathcal{X}} \Pr[X = x] \cdot \log(\Pr[X = x])$$

- Mutual information, a *general* measure of dependence

$$I(X; Y) = \sum_{x \in \mathcal{X}, y \in \mathcal{Y}} \Pr[X = x, Y = y] \\ \times \log \left(\frac{\Pr[X = x, Y = y]}{\Pr[X = x] \cdot \Pr[Y = y]} \right)$$

Information theoretic definitions



Information diagram

2 Test: Kullback-Leibler divergence

$$D_{\text{KL}}(P \parallel Q) = \sum_{z \in \mathcal{Z}} \Pr[Z = z, Z \sim P] \cdot \log \frac{\Pr[Z = z, Z \sim P]}{\Pr[Z = z, Z \sim Q]}$$

Relation to mutual information:

$$\begin{aligned} I(X; Y) &= D_{\text{KL}}(\Pr[X, Y] \parallel \Pr[X] \cdot \Pr[Y]) \\ &= \mathbb{E}_{x \in \mathcal{X}} (D_{\text{KL}}(\Pr[Y|X = x] \parallel \Pr[Y])) \end{aligned}$$

2 Test: F-divergences

$$I_f(P, Q) = \sum_{z \in \mathcal{Z}} \Pr[Z = z, Z \sim Q] \cdot f\left(\frac{\Pr[Z = z, Z \sim P]}{\Pr[Z = z, Z \sim Q]}\right)$$

Different parameter functions f give different measures:

- Kullback-Leibler divergence $f(t) = t \log t$
- Inverse Kullback-Leibler $f(t) = -\log t$
- Pearson χ^2 -divergence $f(t) = (t - 1)^2$
- Hellinger distance $f(t) = 1 - \sqrt{t}$
- Total variation $f(t) = |t - 1|$

1&2: Implicit pdf estimation

Empirical cumulative function:

$$F(x_t) = \frac{1}{n} \sum_{i=1}^n \chi_{x_i \leq x_t}, \text{ where } \chi_{x_i \leq x_t} = \begin{cases} 1 & \text{if } x_i \leq x_t \\ 0 & \text{otherwise.} \end{cases}$$

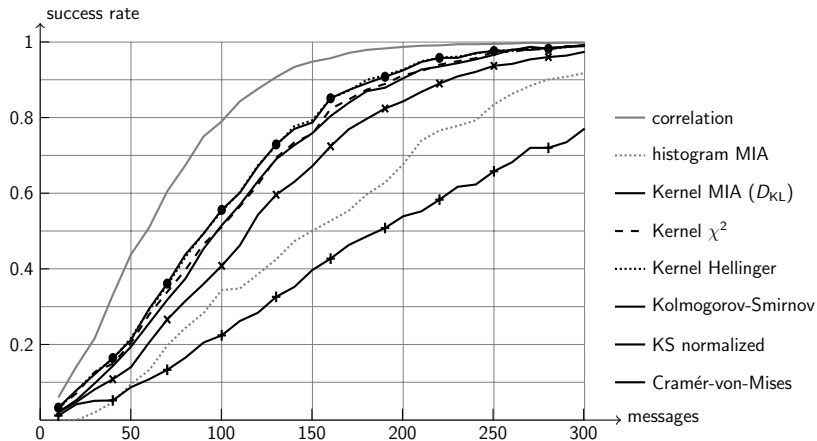
Two sample Kolmogorov-Smirnov test

$$D_{KS}(P||Q) = \sup_{x_t} |F_P(x_t) - F_Q(x_t)|$$

Two sample Cramér-von-Mises test

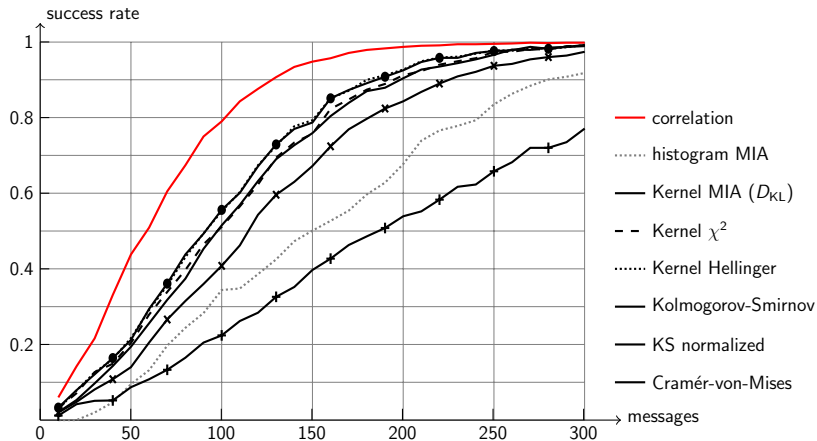
$$D_{CvM}(P||Q) = \int_{-\infty}^{+\infty} (F_P(x_t) - F_Q(x_t))^2 dx_t$$

Experimental results



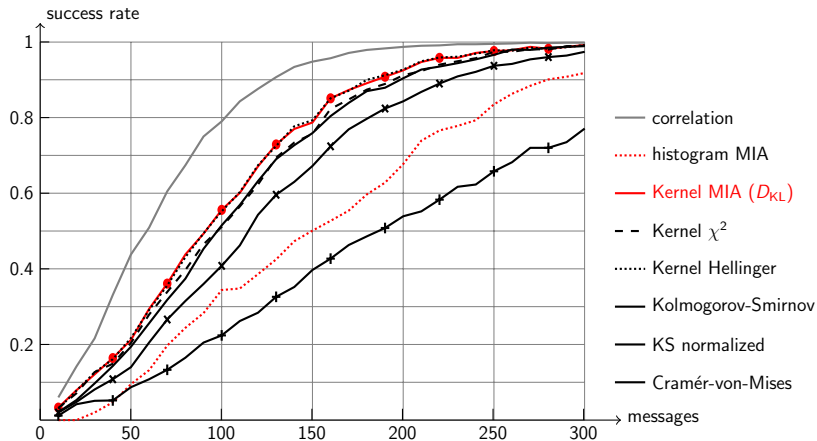
Success rate of different distinguishers

Experimental results



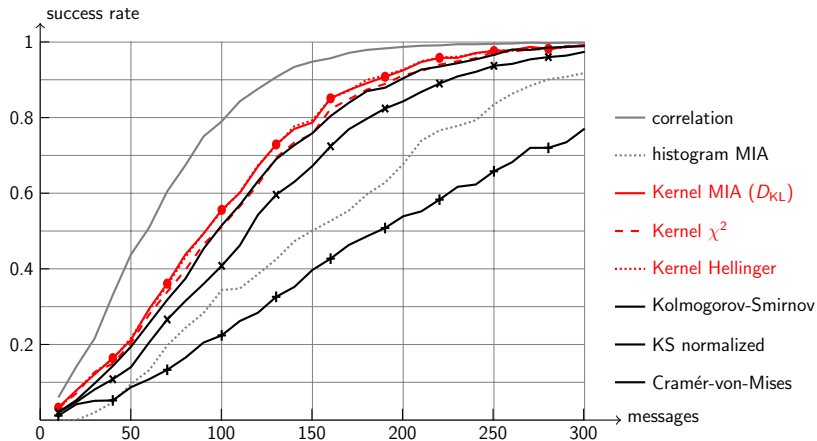
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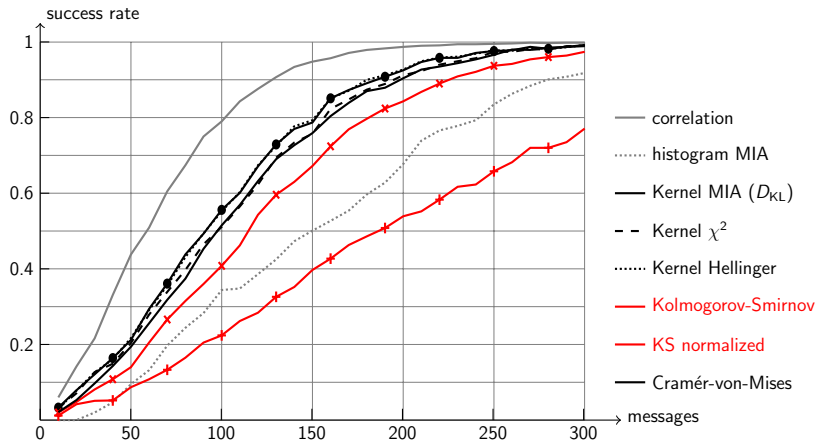
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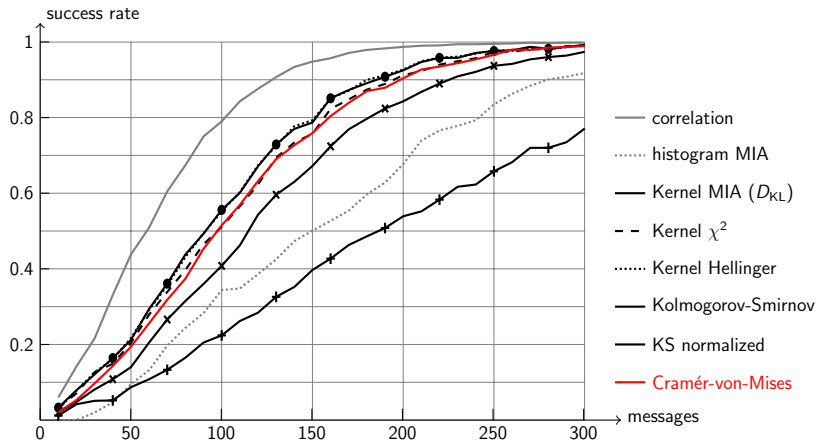
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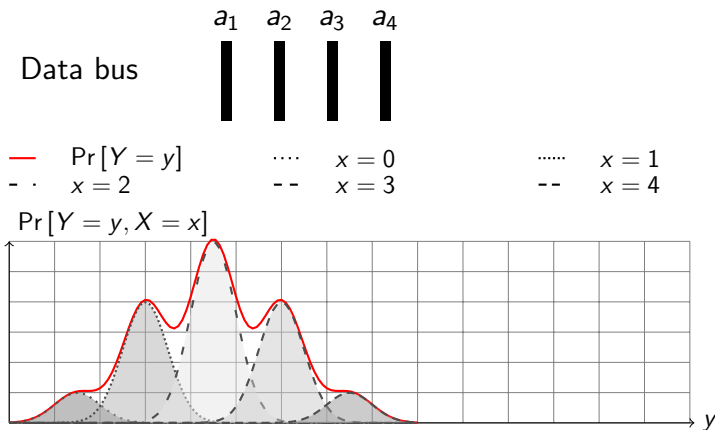


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Outline

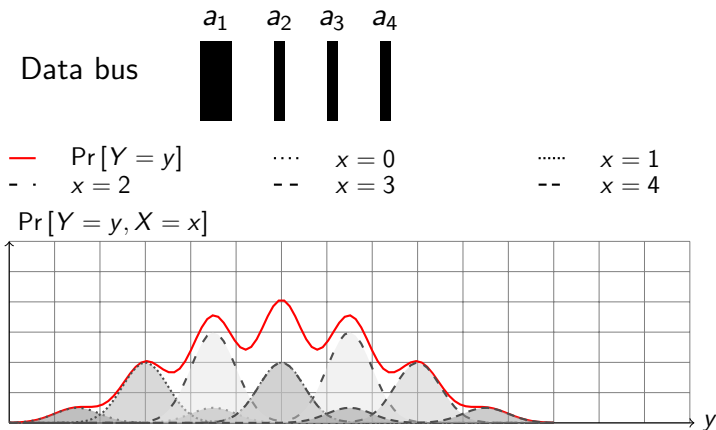
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- 2 **When to use it: MIA versus correlation**
- 3 Why to use it: MIA as an evaluation metric

An example: leaky bit on a data bus



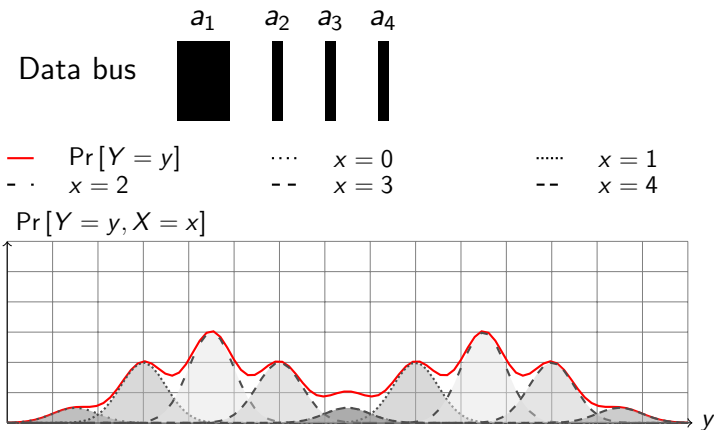
Effect of a leaky bit on the pdfs

An example: leaky bit on a data bus



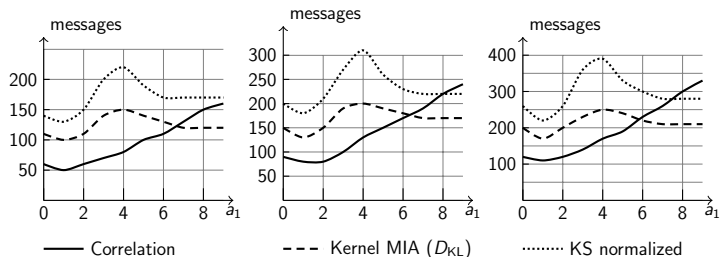
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Effect of a leaky bit on the pdfs

An example: leaky bit on a data bus



Weight of the first leaking bit vs number of messages for a success rate of 50% (left), 75% (middle) and 90% (right)

Limitations

MIA is not the only way to go here: DPA would work!

What about:

- protected logics
- masking scheme

More resilient to erroneous leakage models

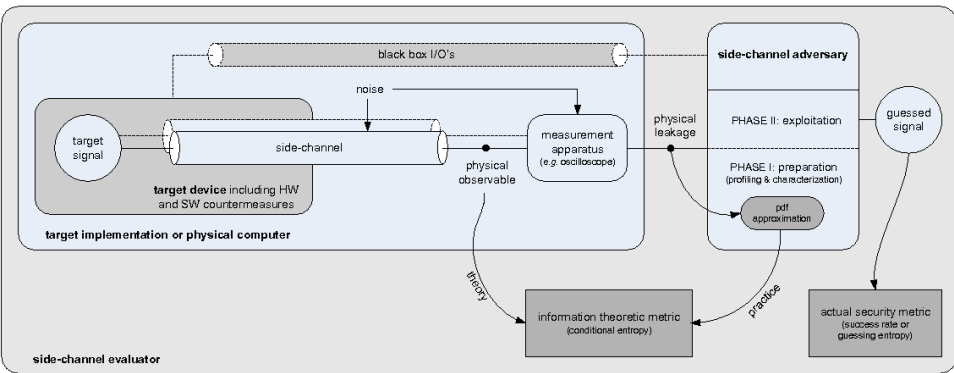
But not immune, requires $I(X_g; Y) > I(X_w; Y)$

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MIA versus Mutual Information Metric

- Eurocrypt 2009:



MIA is not MIM

More precisely:

- 1 MIA: $\hat{I}(X; Y)$ / MIM: $I(K; Y)$
- 2 MIM directly targets the key dependencies
- 3 MIA requires an intermediate variable
- 4 MIM approximates $I(K; Y)$ with “templates”
- 5 MIA estimates $\hat{I}(X; Y)$ “on-the-fly”

→ If the leakage model used by the adversary is not perfect, MIA will underestimate the leakage:

$$I(K; Y) > \hat{I}(X; Y)$$

Summarizing

- MIA is a “toolbox”
- MIA is more resilient to erroneous leakage models
- MIA and MIM are two complementary tools with different purpose: generic adversary and generic evaluation tool

Any Questions?



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