Proﬁling DPA: Efﬁciency and Efficacy
Trade-Offs

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What is profiled DPA? – an overview of the popular methods

What makes a good power model? – our evaluation criteria

How ‘good’ is good enough? – analysis of some example scenarios
SIDECHANELANALALALYSIS*

* (By way of ‘wittily’ acknowledging my frequent pronunciation fails...)

C. WHITNALL (UNIVERSITY OF BRISTOL)  PROFILING DPA  CHES 2013  3 / 20
PROFILING PHASE (SUPERVISED LEARNING)

ATTACK PHASE (CLASSIFICATION)
**Profiled DPA**

**Profiling Phase (Supervised Learning)**

\[ k = k_1 \rightarrow \rightarrow M_{k_1} \]
\[ k = k_2 \rightarrow \rightarrow M_{k_2} \]
\[ \vdots \]
\[ k = k_n \rightarrow \rightarrow M_{k_n} \]

**Attack Phase (Classification)**

C. Whitnall (University of Bristol)
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\[ k = ? \]
**Two Typical Methods**

### 'Classical' templates:
- Separate multivariate Gaussian models for each key-dependent value
- Covariance matrix estimated for each key-dependent value

### Linear regression-based templates:
- Linear regression model fitted to the pooled data at each time point
- Covariance matrix estimated for pooled data (2\textsuperscript{nd}, independent sample)

Choose the key hypothesis which maximises the log-likelihood of the observed traces.

**OR (ignoring noise):**
Choose the key hypothesis which maximises the correlation between the model fitted values and the observed traces.
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Consider an 8-bit intermediate value target (e.g. AES S-box output)...

- Classical templates have *fixed complexity*: $2^m$ conditional mean vectors, $2^m$ covariance matrices.

- Linear regression has *adjustable complexity*: an intercept, coefficients on all the equation terms, and one covariance matrix.
  - Potentially large reduction in profiling traces needed (e.g. linear model expression requires only $m + 1$ coefficients).
  - Potentially substantial degradation in model quality if simplifying assumptions are not correct.
  - Higher-order terms in the model equation militate against model degradation but add to profiling data complexity.

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Previous Work


- LR templates recover key with fewer (profiling) traces but classical achieve higher success rates once profiling sample is large.
- Analysis primarily experimental: true distributions unknown so difficult to comment on model quality.
- Tested scenarios limited and favourable to LR (close to HW).


- Information theoretic metric can be used to quantify model quality.
- Analysis geared more towards theory (establishing an evaluation framework).
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Our Contribution

- Explore trade-offs in a wider range of scenarios, including those not well-suited to low-degree approximations.
- Theoretic (rather than experimental) evaluation where possible.
- Hypothetical scenarios with fully-specified leakage distributions give concrete benchmarks for model quality/performance.
What Makes a Good Power Model?

1. **Profiling complexity**: the fewer traces needed to build the model, the better.

2. **Goodness-of-fit**: the closer the model is to the actual leakage distribution, the better.

3. **DPA performance**: the fewer the traces needed to recover the key from the target device, the better.
Difficult to measure theoretically: sample size formulae exist for simpler statistical problems but not for precise coefficient estimation.

Empirical approach:
- 1,000 repeat experiments on randomly drawn balanced samples
- Gaussian noise at high (8) medium (1) and low (0.125) signal-to-noise ratios
- Fit models of degree ranging from 1 through to 8
- Count number of traces required to reach a certain threshold of precision
Measuring Goodness-of-Fit

Find least squares solution \( \{\hat{\beta}_0, \ldots, \hat{\beta}_p\} \) for the system of equations representing the regression in the absence of noise:

\[
\{Y_v\}_{v \in V} = \left\{ \sum_{j=0}^{p} \beta_j g_j(v) \right\}_{v \in V}
\]

Compute coefficient of determination – proportion of variation in the leakage function which is accounted for by the model:

\[
\rho \left( \left\{ \sum_{j=0}^{p} \hat{\beta}_j g_j(v) \right\}_{v \in V}, \{Y_v\}_{v \in V} \right)^2
\]
Measuring DPA Performance

- Compute the theoretic correlation distinguishing vector under each model:

\[
D_\rho(k) = \rho(Y, M_{LR}(V_k)) = \frac{\text{cov}(Y, M_{LR}(V_k))}{\sqrt{\text{var}(Y)} \sqrt{\text{var}(M_{LR}(V_k))}}
\]

- Use sample size formulae to calculate the number of traces required to distinguish the true key from the nearest rival:

\[
N^* = 3 + 8 \cdot \frac{z^2_{1-\alpha}}{\left(\ln \frac{1+D_\rho(k^*)}{1-D_\rho(k^*)} - \ln \frac{1+D_\rho(k^{nr})}{1-D_\rho(k^{nr})}\right)^2}
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\[ \alpha: \text{“significance level”} \]

Quantile of the standard normal \( N(0, 1) \)
Consider leakage of the form $L(v) + \varepsilon$, where $L(v)$ is the deterministic, data-dependent component which we will call the \textit{leakage function} and $\varepsilon \sim N(0, \sigma_\varepsilon)$ is \textit{additive Gaussian noise}. (The intermediate value $v$ in our analysis is the AES S-box output.)

1. The leakage function is proportional to the \textit{Hamming weight}, as motivated by typical behaviour of CMOS technology.

2. Adjacent wires interact so that the leakage is proportional to the \textit{Hamming weight plus quadratic terms} involving adjacent bits of the intermediate value.

3. The leakage is a \textit{highly nonlinear} function of the intermediate bits such as that arising from hardware implementations of AES.
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1. The leakage function is proportional to the Hamming weight, as motivated by typical behaviour of CMOS technology.

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3. The leakage is a highly nonlinear function of the intermediate bits such as that arising from hardware implementations of AES.
- Affects all leakage scenarios similarly.
- Sample sizes to estimate maximum degree polynomials are around 30 times more than those to estimate linear polynomials.
- Little change in complexity between degree 6 and degree 8 models.
- Reasonable savings only possible at degree 5 or lower.
- Sample size increases as signal decreases but relationship between models of different degree is consistent.
Hamming weight leakage:
- Perfectly approximated by a linear model function.
- Performs equivalently to ‘classical’ models.
Low Degree Leakages

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Leakage with adjacent interactions:
- Closely approximated by a linear model function.
- Performance only marginally diminished.
Toggle Count-Based Leakage:

- Linear model inadequate to approximate the leakage – captures just 6% of the variation.
- Degree 4 model accounts for about two thirds of the variation, with less than half the number of parameters required for the classical model.
**Toggle Count-Based Leakage:**

- Very little difference in distinguishing power between the degree 5 and classical models.

- Linear and quadratic models are able to recover the key, but by very small margins and requiring lots of traces – over a hundred times as many in the case of the linear model.

- Degree 4 model requires around twice as many traces.
Experiments suggest the formula overstates the sample size in the case of highly-degraded models (further work needed).

<table>
<thead>
<tr>
<th>Model</th>
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<th>Profiling complexity</th>
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<tr>
<td></td>
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<td>HW</td>
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- Linear regression is an excellent alternative to classical profiling when the true leakage function is simple.

- Over-simplified assumptions when the leakage is complex can substantially diminish attack performance.

- Device evaluation perspective:
  - Classical profiling remains the best way to test for vulnerability against the strongest possible adversary.

- Attacker perspective:
  - In our example, degree 4 models offer a promising trade-off between profiling and attack complexity.
  - Even minimal profiling can substantially increase attack performance relative to standard assumptions (such as Hamming weight leakage) when those assumptions do not hold.
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Thank you for listening!

Any questions?